Round Robin: A Fair Tournament System

Keywords: combinatorics, probability and statistics, combinatorics

Imagine you're organizing a school tournament in table tennis, chess, e-sports, or futsal. You want it to be as fair as possible – so that every player has the chance to face all the others. That's exactly what the round robin system is for.

Its main advantage is fairness: the final ranking depends solely on the players' or teams' performance, not on a random draw of opponents. On the other hand, the number of matches grows quickly with the number of participants – planning such a tournament can be quite a challenge. And this is where combinatorics comes into play – the mathematics of counting possibilities.

Futsal Tournament

Exercise 1. Nine teams have registered for a futsal tournament. It will be played in a round robin format, meaning each team plays one match against every other team. For each win, a team earns 2 points; for a draw, 1 point; and for a loss, 0 points. The final ranking is determined by the total number of points earned across all matches.
How many matches need to be played in the tournament? In how many different ways can the tournament schedule be arranged, assuming there is only one field available and the matches are played one after another?

Exercise 2. Show that if any team in the tournament described in the previous problem earned a total of 13 points, then it must be among the top four teams in the tournament.

A Fairer Tournament

This time, seven teams have signed up for the next edition of the futsal tournament described in the previous problems. When preparing the tournament schedule, the organizer introduced a new condition: no team is allowed to play in two back-to-back matches. This way, players can avoid playing while tired, and the tournament becomes fairer.

Libor came up with an algorithm for generating a sequence of matches that meets this requirement. His idea is based on the following table:

The portal Math4U is developed within the international projects of the Department of Applied Mathematics (VSB-Technical University of Ostrava) and other institutions in Czechia, Poland, Slovakia, and Spain.

Funded by the European Union

	tým 1	tým 2	tým 3	tým 4	tým 5	tým 6
tým 2	D_1					
tým 3	D_2	D_1				
tým 4	D_3	D_2	D_1			
tým 5	D_4	D_3	D_2	D_1		
tým 6	D_5	D_4	D_3	D_2	D_1	
tým 7		D_5	D_4	D_3	D_2	D_1

Figure 1: Table for generating a fair tournament schedule

Each cell in the i-th row and j-th column corresponds to the match between team $(i+1)$ and team j. The desired match sequence will follow the order in which Libor selects these cells. For clarity, we will label the cells according to the teams involved in each match, such as $[1;2]$, $[3;5]$, and so on. Next, we define the longest diagonal starting at $[1;2]$ and ending at $[6;7]$ as D_1, the shorter diagonal starting at $[1;3]$ and ending at $[5;7]$ as D_2, and so on.

Libor's algorithm proceeds as follows: - First, select the cell in the first column and last row, i.e. $[1;7]$; - Next, go through all the cells of diagonal D_1 that lie in even-numbered columns, from left to right; - Then go through the remaining cells of diagonal D_1 that lie in odd-numbered columns, again from left to right; - Then go through all the cells of diagonal D_2 from left to right; - Then do the same for diagonal D_3, then D_4, and so on.

For a tournament with seven teams, this gives the following match sequence:

$$\begin{array}{ccccccc}
[1;7], & [2;3], & [4;5], & [6;7], & [1;2], & [3;4], & [5;6], \\
[1;3], & [2;4], & [3;5], & [4;6], & [5;7], & [1;4], & [2;5], \\
[3;6], & [4;7], & [1;5], & [2;6], & [3;7], & [1;6], & [2;7].
\end{array}$$

Exercise 3. Using Libor's algorithm, list the sequence of matches for a tournament with 9 teams, and verify that no team appears in two consecutive matches.

Exercise 4. Does Libor's algorithm work for any number of participating teams? If not, for which values of n does it work? And can you construct the required sequence for those cases?

The portal Math4U is developed within the international projects of the Department of Applied Mathematics (VSB-Technical University of Ostrava) and other institutions in Czechia, Poland, Slovakia, and Spain.

Funded by the European Union