

# Medical Station at a Running Race

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Imagine you're helping organize a large running race. Dozens of runners are about to hit the track, and your task is to decide where to place the medical station so that it is as helpful as possible. Should it be close to the starting line? Or perhaps somewhere in the middle? And what if there are several checkpoints along the route? Where is the best spot from which everything is roughly the same distance?

It might sound simple at first, but when you think about it more carefully, you'll realize that finding the best possible location isn't so easy after all. In the following exercises, we'll explore this kind of situation. And who knows — maybe thanks to us, a runner will safely make it to the finish line.

**Exercise 1.** Along a 45 km race course, there are three checkpoints, and a medical station needs to be placed somewhere on the route. The first checkpoint is located at the 13th kilometer, the second at the 26th kilometer, and the third at the 37th kilometer. Because the medical station should be as close as possible to the checkpoints, the start, and the finish, the race organizer wants to place it so that the sum of distances from the medical station to these five locations is as small as possible. At which kilometer mark should the medical station be placed? Is this the only possible location the organizer should choose? Assume the race finishes at a different location than where it starts, and that there is no shorter path between any points than along the race route itself.

**Exercise 2.** How does the solution to the previous exercise change if there are four checkpoints located at the 17th, 30th, 35th, and 40th kilometers?

This problem can also be generalized.

**Exercise 3.** There are  $n$  different checkpoints placed along the race course. Where should the medical station be located so that the sum of its distances from all the checkpoints, the start, and the finish is as small as possible?