Anamorphosis

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Anamorphosis in Visual Art

Anamorphosis is a type of visual trick or art in which a hidden image is revealed when viewed from a specific angle. Anamorphosis relies on the observer finding the correct spot from which to look. This kind of art has a long and rich history. One of the most famous paintings using anamorphosis is The Ambassadors (1533) by the German painter Hans Holbein the Younger (1497 - 1543).

Figure 1: The Ambassadors

At the bottom of the painting, there is a strange elongated object. You can only make out what it is if you stand against the wall near the right frame of the painting and look from that direction. If you find the right position from which to look, you will see that it is a skull.

Anamorphic art can also use reflections of paintings or sculptures in a cylindrical mirror[1].

By the end of the 20th century, anamorphic art experienced a major revival in photography, drawing, and large-scale installations. Some artists create anamorphic images from everyday objects such as electronics, shoes and socks[2]. Anamorphoses also appear in street art. These are often drawings on

[1] https://commons.wikimedia.org/wiki/File:Anamorphic_frog_sculpture_by_Jonty_Hurwitz.jpeg.
[2] https://www.youtube.com/watch?v=y__zPc3MZm4.

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sidewalks, streets, or walls that surprise and momentarily confuse passers-by. For example, it can be a drawing that looks like a hole in the ground, into which a fall is imminent, legs sticking out of a wall or a canal, etc. Anamorphoses based on central projection are more convincing when viewed with one eye or through a camera lens. However, if the center of projection is far enough from the object, if the shading is done well, or if the surrounding environment supports the illusion of space, the effect becomes even more realistic.

Practical Applications

In the film industry, anamorphic lenses are sometimes used to shoot movies. They were originally designed so that wide-format images would fully utilize the area of standard 35mm film frames. Without them, widescreen images would leave the top and bottom parts of the frame unused. Despite the arrival of high-resolution digital sensors, anamorphic lenses are still used today for the uniqueness of the resulting image.

Some cities introduced pedestrian crossings that at some point looked like levitating prisms from the perspective of oncoming drivers. After a short trial period, these crossings were mostly removed, as drivers tended to brake too sharply when approaching them.

The technique of anamorphic projection can also be seen in some sports stadiums, where it is used for advertising. Company logos are painted directly onto the playing field, and from the angle of the TV camera, the text appears to be standing vertically on the surface.

Anamorphoses of Basic Solids

In the following text and examples, we will create anamorphoses of basic solids using central projection onto a plane. The plane in which we will draw these anamorphic images is called the projection plane. In our case, the projection plane will be a sheet of paper we draw on, which naturally limits the size of the objects we can create. We will then observe the resulting images through the camera eye via a mobile phone or camera. If you have the opportunity, you can create anamorphic images outdoors, ideally away from roads or traffic.

Pyramid and Cone

Perhaps the easiest solids to create anamorphic images of are the pyramid and the cone—provided their bases lie in the projection plane. Let's explain the principle using a pyramid. In addition to the solid itself, we also need to define the center of projection S and its perpendicular projection onto the projection plane S_1. You can think of the center of projection as the observer's eye, and the perpendicular projection as the spot where the observer is standing. The distance $S_1S = d$ is then the distance from the projection plane to the center of projection. For a regular square pyramid, we denote its apex as V and the perpendicular projection of the apex onto the projection plane as V_1. The intersection of the line SV (called the projection ray) with the projection plane is then obtained as the intersection of lines SV and S_1V_1 (see the following figure on the left). It's useful to sketch such a diagram when thinking about how the illusion works and what the central projection will look like. However, this spatial picture is not needed to determine the anamorphosis of the pyramid.

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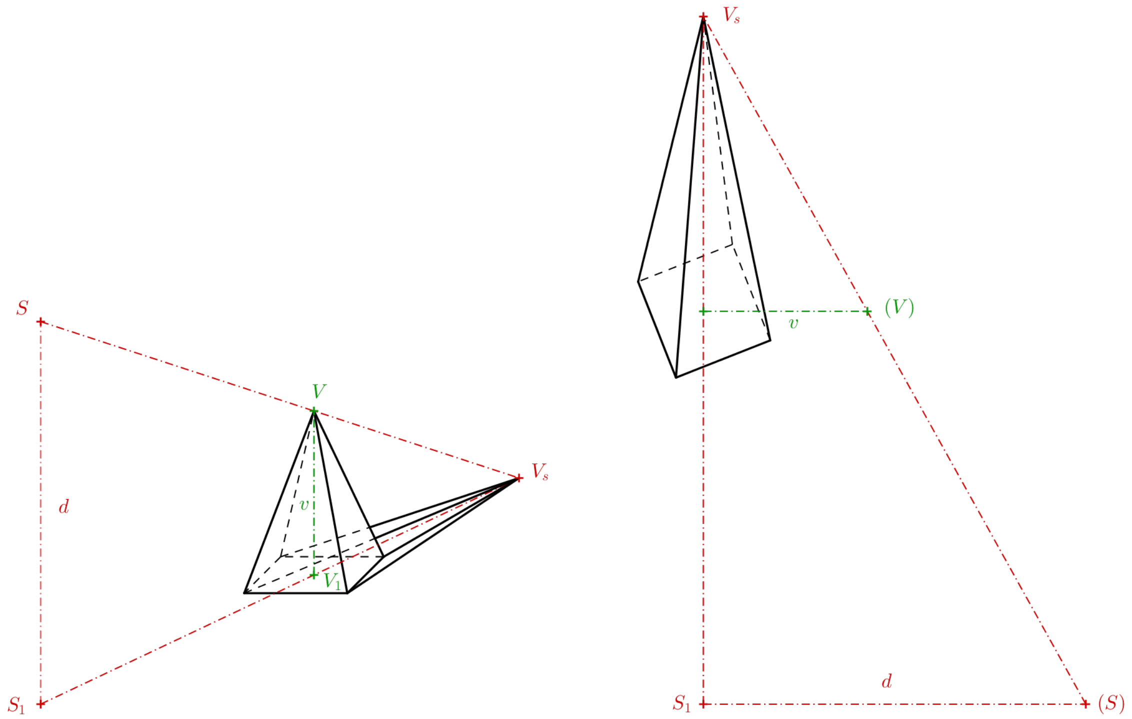

Figure 2: Anamorphosis of the pyramid

What we actually need is just the trapezoid S_1V_1VS, which can also be represented in the projection plane as the trapezoid $S_1V_1(V)(S)$ (see previous figure on the right). The points that were previously in space outside the projection (points V and S) are now shown in parentheses in the projection to distinguish them. The points (V) and (S) were created by rotating the plane S_1VS by $90°$ into the projection around the line S_1V_1. If we know the height of the pyramid, the distance of the observer's eye to the projection plane, and the distance S_1V_1, we can draw the trapezoid. By extending its non-parallel sides, we find the point V_s.

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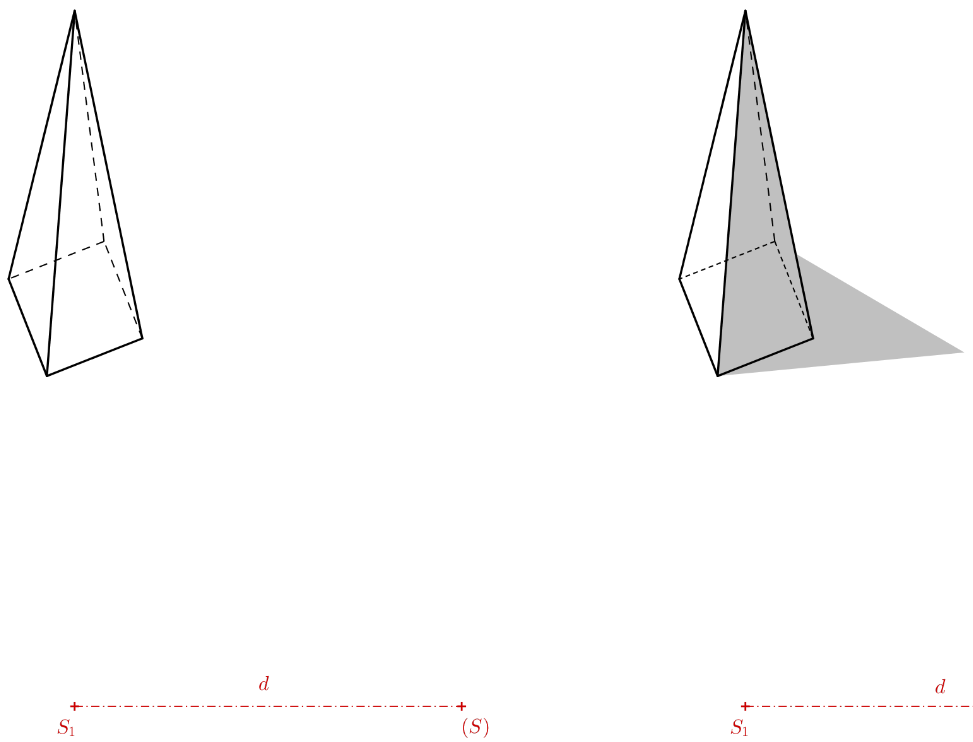

Figure 3: Anamorphosis of the pyramid

The result (see the previous figure on the left) should be drawn without construction lines. We can then observe it through the lens of a camera. When viewed through the camera, we find that the hidden edges of the bottom base are best drawn with a denser dashed line than the projection of the invisible side edge. To make the pyramid appear more realistic, we can shade the image. The shadow can be estimated freely, and the cast shadow of the apex can be chosen as needed. The anamorphosis of the pyramid is complete. For the illusion to work, the camera lens must be positioned directly above point S_1 at a height equal to the distance $S_1(S)$. As seen through a camera, the final image should resemble the one shown in the following figure.

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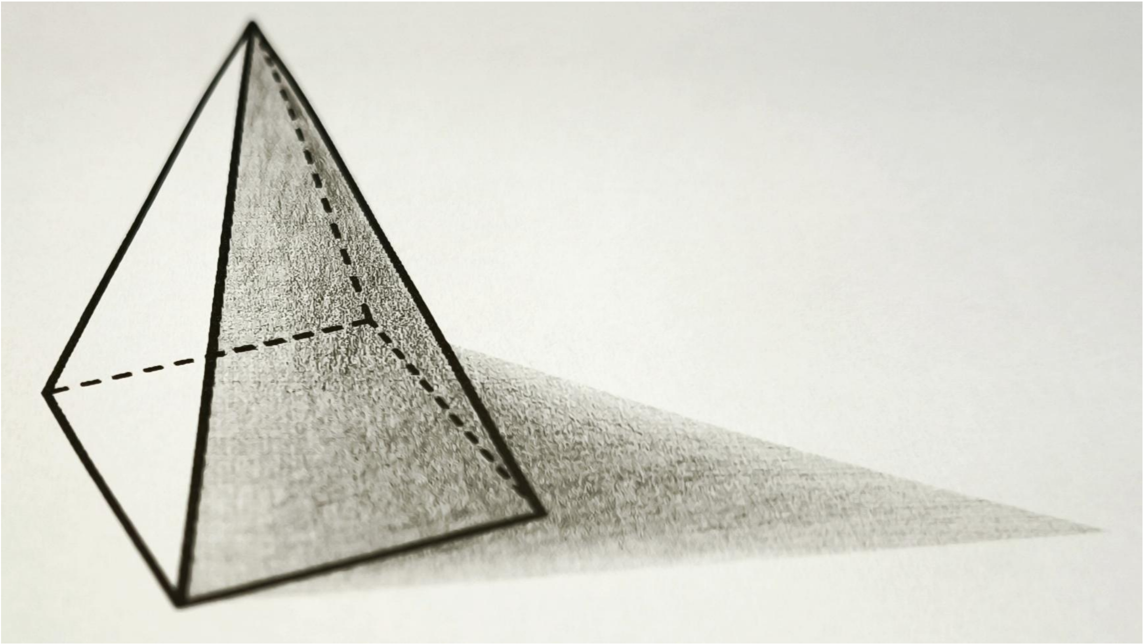

Figure 4: Anamorphosis of the pyramid, camera view from the center of S

Exercise 1. We want to draw a shape on the ground that, when viewed in space, appears as a cone with a height of $1\,\mathrm{m}$ and a base with a radius of $r = 0.4\,\mathrm{m}$. As before, we denote the center of projection by S and its perpendicular projection onto the ground by S_1. We assume that the eye of an average observer is at a height of $150\,\mathrm{cm}$ above the ground. At what distance must V_s be from V_1 (V_s is the central projection of the cone's apex onto the projection plane, V_1 is the perpendicular projection of the cone's apex onto the projection plane), assuming the observer is standing 3 meters away from point V_1?

Exercise 2. We are given a base circle k with center V_1 and point V_s (see figure below for reference). Imagine a right circular cone in space with the base circle k and a center of projection S, such that V_s is the central projection of the cone's apex. Point V_1 is the perpendicular projection of the cone's apex onto the projection plane (i.e., the paper). Determine the outline of the central projection of the cone.

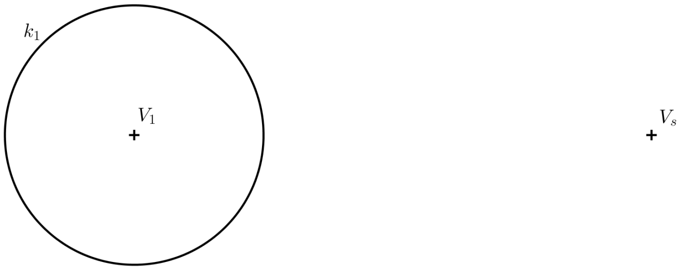

Figure 5: Exercise 2 assignment

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Exercise 3. To solve the previous problem, determine the position of point S (using S_1 and (S)), if the height v of the cone in space and the distance $d = |S_1S|$ are known. See the following figure for reference, the lengths of v and d are given as segments.

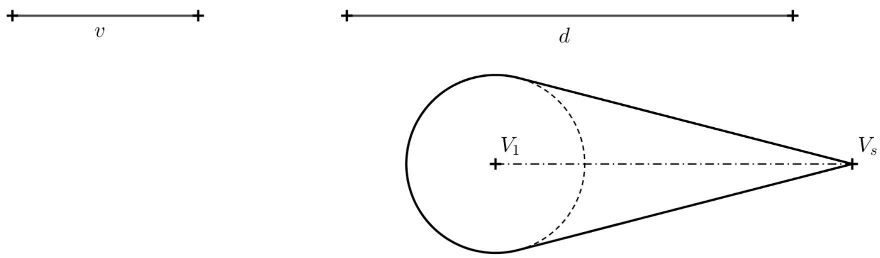

Figure 6: Exercise 3 assignment

Prism and Cylinder

To construct an anamorphic image of a prism or a cylinder, we use homothety. Let's explain why, using the example of a cube shown in the diagram.

There is a similarity relationship between the top face of the cube and its projection in space, with the center of similarity at point S (this follows from the similarity of triangles). Since the bottom face of the cube is also the perpendicular projection of the top face onto the projection plane, there is a similarity relationship between the bottom face and the central projection of the top face, with the center of similarity at point S_1

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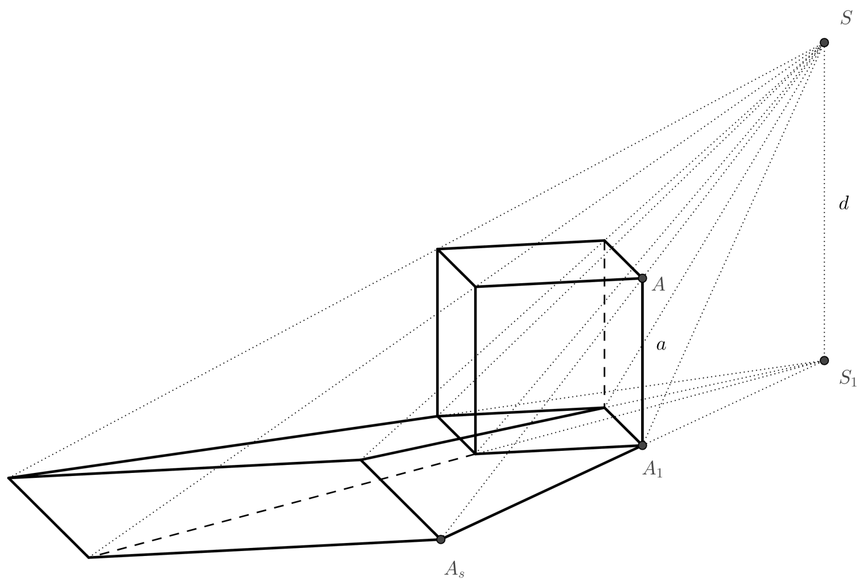

Figure 7: Anamorphosis of a cube

Exercise 4. Determine the anamorphic projection of a cube. The square base is given by two opposite vertices A_1 and C_1. The position of the point S_1 (the perpendicular projection of the center of projection S) is also given. The length d is given by the radius of the circle k.

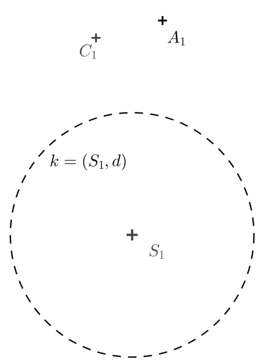

Figure 8: Exercise 4 assignment

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Exercise 5. We are given two circles of different sizes (see the figure below for reference). Determine their center of homothety S_1 that maps one circle on the other, and draw their common tangents so that the resulting image is an anamorphosis of a cylinder.

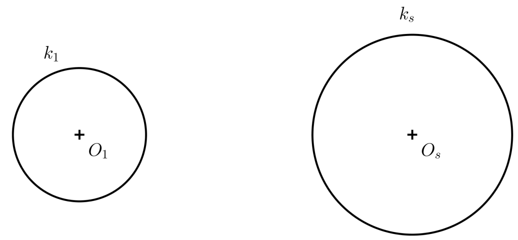

Figure 9: Exercise 5 assignment

Exercise 6. The ratio of homothety $H(S,k)$ in the previous exercise, which maps point O_1 to O_s, is $k = 1.5$. What must be the ratio $d : v$, where $d = |S_1S|$ and $v = |O_1O|$ (the height of the imaginary cylinder in space), in order for the spatial illusion to work?

References and literature

Literature

https://en.wikipedia.org/wiki/Anamorphosis

Image Sources

- The Ambassadors https://en.wikipedia.org/wiki/File:Hans_Holbein_the_Younger_-_The_Ambassadors_-_Google_Art_Project.jpg
- Skull (detail of the painting The Ambassadors viewed from the correct location) https://en.wikipedia.org/wiki/File:Holbein_Skull.jpg

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