Math4You

2023–2025

Miura-ori

Miura-ori (Miura fold) is one of the most famous ways of folding paper in origami. When we hear the word origami, we probably first imagine a paper model in the shape of an animal or a boat. These simple shapes often serve as the first step into the fascinating world of paper folding. But there are also origami that take several hours or even days of skilful work.

What is fascinating, is that the principle of folding as we know it from origami is also found in the nature around us and in ourselves. A flower's bud gradually opening up, the gently unfolding wings of insects hidden under elytron (hardened forewings), the complex structure of human DNA, or the walls of the large intestine, all use principles similar to those found in origami. The ability of materials and structures to "fold" and "unfold" as needed is one of the basic building blocks of life. In recent years, origami has experienced a real boom, not only as a hobby for paper lovers, but also in many artistic and scientific fields. It has influenced architects, furniture designers, artists and scientists. Origami has thus transformed from a mere art of paper folding into a tool that helps shape the future.

Origami in science

One of the areas where origami has found application is the space program. When transporting large objects in space, it is necessary for them to be foldable to a smaller sizes. One such object is the starshade, a giant shield designed to block out the light of a star. Astronomers need it to observe planets that are near bright stars, as the brightness of these stars makes observation impossible.

Using origami in robotics is also advantageous. Robots designed using origami have the potential to be faster and cheaper to manufacture than robots created using traditional techniques. They are also cheaper and easier to manufacture than robots created using traditional techniques.

New types of material incorporating origami structures are emerging. The patterns of these origami structures are often based on the periodic division of a plane. In the following problems, we will examine the most well-known pattern and method of paper folding.

The usage of Miura-ori and its folding technique.

This folding technique was invented by Japanese astrophysicist Koryo Miura. When the structure is opened, it appears to be periodically divided into rows of parallelograms.

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This particular pattern can be opened or closed in one simple way. Simply pull on one corner to unfold the origami with minimal effort. Miura designed this folding method for solar panels. In 1995, a solar panel featuring this design was deployed on the Japanese Space Flyer Unit satellite. Since then, this folding technique has found many other applications, including portable solar panels and foldable bulletproof shields for police forces. In Japan, maps are also folded this way to avoid wear and tear on the corners.

The pattern is also used in materials engineering as the inner part of a sandwich structure. When made of Kevlar paper, fiberboard, or plastic film and sandwiched between two cover sheets, it creates a lightweight structure that is very strong and stable.

Let's try to fold this pattern. We can start with the usual paper format $A4$, which has dimensions 210 mm x 297 mm. For smooth opening and closing of the pattern, it is advisable to divide the sides into an odd number of sections. We start by dividing the shorter dimensions of the $A4$ format into 5 equally large sections, so one section will have a length of 42 mm. We will connect points lying opposite each other and fold the paper in the shape of an accordion in these lines.

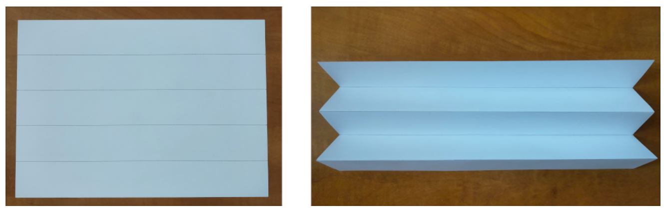

Figure 1: Folding the Miura Ori pattern

Now divide one of the longer edges into 7 parts. Through any division point, draw a line that forms a non-right angle with the longer edge. Through the other division points, draw parallels to this line.

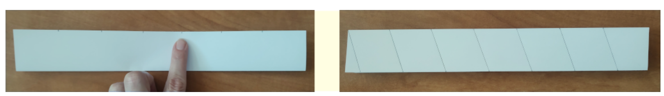

Figure 2: Folding the Miura Ori pattern

We fold the accordion in these mutually parallel lines. Now we have all the necessary folds of the resulting pattern, but some are bent in a different direction than we need.

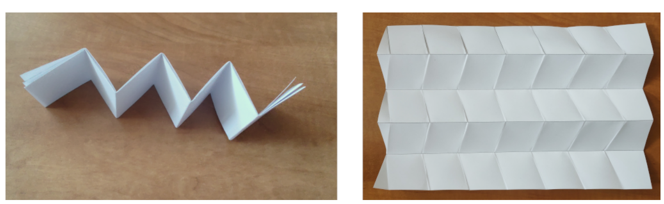

Figure 3: Folding the Miura Ori pattern

We unfold and rearrange the accordion so that the individual broken lines become ridges and valleys in an alternating manner.

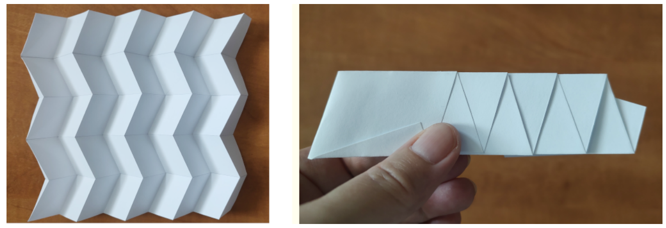

Figure 4: Folding the Miura Ori pattern

The folds of the Miura Ori pattern are indicated in origami as in the following image, i.e. ridges are solid, valleys are dashed.

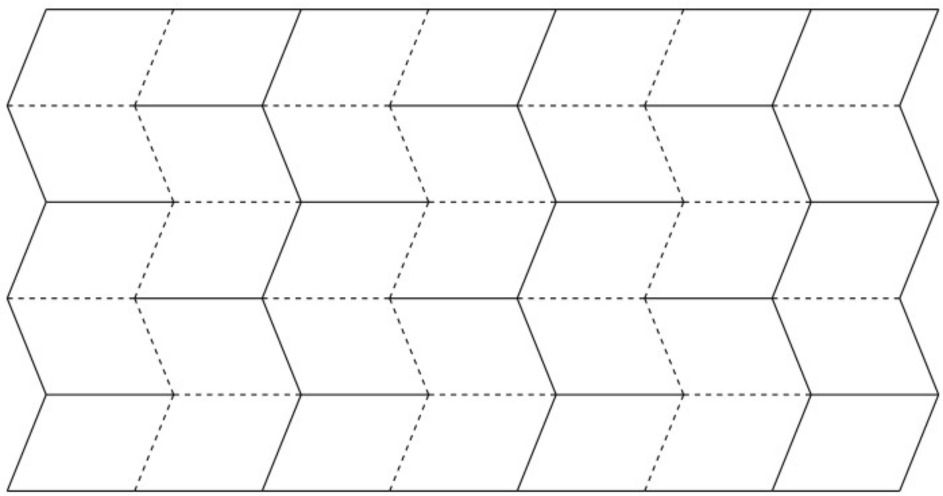

Figure 5: Miura Ori

Exercise 1. The figure below shows two versions of what the final pattern can look like when folded (using the same initial paper size and number of divisions). On what parameters does the resulting length depend?

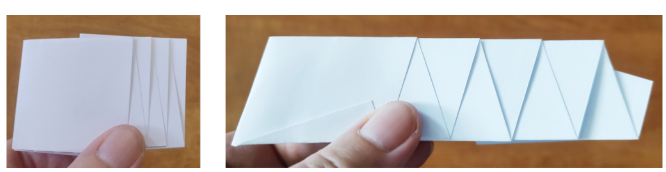

Figure 6: Different solutions

Solution. It is clear from a simple comparison of the patterns that that the length of the resulting pattern depends on the angle. at which the lines form a non-right angle with the longer side. This is obvious because it is the only factor that makes the patterns different. Let us denote this acute angle by α. It holds that if you make the angle α more acute, the length of the folded pattern will be greater. The closer this angle is to $90°$, the shorter the folded pattern.

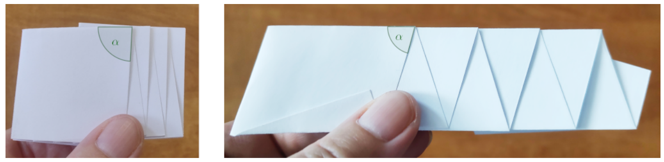

Figure 7: Different solutions

So far we have compared two different results for similar division. But how exactly does the length of the folded pattern depend on the angle α and other parameters? When investigating this dependence further, it is a good idea to focus on the fundamental part of the pattern.

Exercise 2. How does the length x of the fundamental part of the Miura-ori pattern in its folded position depend on the size of the angle and the lengths d and l? See the figure for the input data.

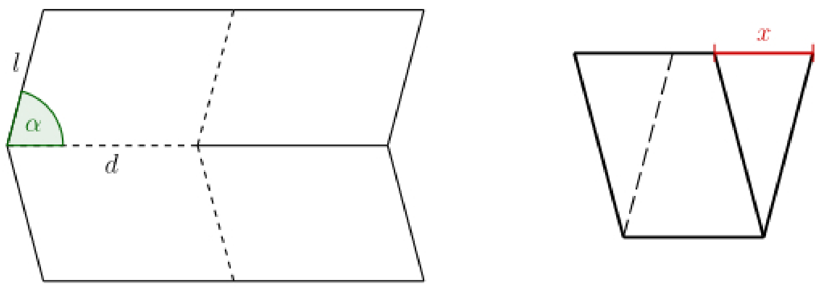

Figure 8: Fundamental part of Miura-ori in unfolded and folded version

Solution. It is important to mark the parameters in the image in appropriate places.

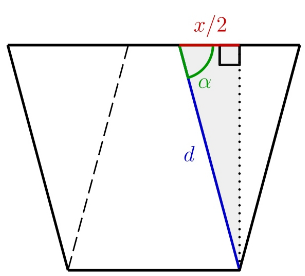

Figure 9: Finding the length

For $\cos\alpha$ we then have

$$\cos\alpha = \frac{\frac{x}{2}}{d} = \frac{x}{2d},$$

from which we simply express

$$x = 2d \cdot \cos\alpha.$$

The length x therefore does not depend on l at all, but only on the angle α and the length d.

Literature and references

- https://www.youtube.com/watch?v=uFyJykl1O0k
- https://www.youtube.com/watch?v=ZVYz7g-qLjs
- https://laughingsquid.com/millidelta-small-origami-inspired-robot/
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