Which Lottery Ticket Is More Profitable?

In life we very often encounter situations that involve chance and probability. Imagine being faced with a choice between several options—for example, when choosing a lottery ticket or investing in a project. Each choice has its risks and potential rewards, but how can we determine which one is the most advantageous? This is where the so-called *expected value* comes into play.

Expected value tells us the average outcome we can anticipate when choosing a particular option. It helps us better estimate which option is likely to pay off in the long run. It is not an exact prediction, but a tool that allows us to better understand risk and reward, both in simple games and in real life decisions.

Let's consider two lottery tickets, for example:

- Ticket A: It costs 10 CZK and has a 0.1 probability of winning 100 CZK; otherwise, it wins nothing.
- Ticket B: It costs 10 CZK and has a 0.2 probability of winning 60 CZK; otherwise, it wins nothing.

For ticket A, we expect that if we buy 10 tickets, one of them will win 100 CZK while the reaming nine will win nothing. Therefore, we can expect that each lottery ticket will yield an average return of 10 CZK.

Similarly, for lottery ticket B, we expect that if we buy 10 tickets, two of them will win 60 CZK and eight will win nothing. We can therefore expect each lottery ticket to yield an average return of 12 CZK.

This shows that ticket B is the better option.

Expected Value

The average win we just calculated is called the *expected value*.

In general, we can say that for a random variable X that takes on finitely many values $x_1, \ldots, x_k$ with probabilities $p_1, \ldots, p_k$, we calculate its expected value using this formula:

$$EV = \sum_{i=1}^{k} x_i p_i.$$

Which Lottery Ticket Is the Best?

Let's take a look at three lottery tickets. The 50 CZK Black Pearl ticket, the 100 CZK Black Pearl ticket and the Rental King lottery ticket worth 50 CZK.

The prize structure for the 50 CZK Black Pearl lottery tickets, of which there are 13,000,000 in total, is as follows.

The portal Math4U is developed within the international projects of the Department of Applied Mathematics (VSB-Technical University of Ostrava) and other institutions in Czechia, Poland, Slovakia, and Spain.
Funded by the European Union

Prize amount (in CZK)	Number of winning tickets
50	1,820,000
100	1 040,000
150	260,000
200	130,000
300	130,000
500	104,000
1,000	5,550
2,000	2,300
4,000	480
10,000	185
20,000	84
100,000	14
1,500,000	6
Total	3,492,619

The prize structure for the 100 CZK Black Pearl lottery ticket looks similar, with a total of $15{,}000{,}000$ issued tickets.

Prize amount (in CZK)	Number of winning tickets
100	2,400,000
200	900,000
300	450,000
500	150,000
600	150,000
900	75,000
1,000	75,000
1,500	20,000
6,000	4,000
20,000	185
50,000	84
100,000	30
200,000	13
5,000,000	6
Total	4,224,318

Last but not least, let's take a look at the Rental King lottery ticket, with a total of $8{,}000{,}000$ tickets issued. The prizes are shown in the table below.

Prize amount (in CZK)	Number of winning tickets
50	960,000
100	720,000
150	160,000
250	160,000
500	70,000
1,000	1,300
2,000	500
5,000	160

The portal Math4U is developed within the international projects of the Department of Applied Mathematics (VSB-Technical University of Ostrava) and other institutions in Czechia, Poland, Slovakia, and Spain.
Funded by the European Union

Prize amount (in CZK)	Number of winning tickets
10,000	80
100,000	6
3,500,000	3
Total	2,072,049

The top prize of $3{,}500{,}000$ CZK is not paid at once, but consists of an immediate prize of $500{,}000$ CZK and an annuity of $50{,}000$ CZK for 5 years.

Exercise 1. Which ticket has the highest chance of winning?

Exercise 2. What is the expected value of each ticket?

Exercise 3. In the previous examples, we considered the top prize of the Rental King lottery to be $3{,}500{,}000$ CZK. But is this really the actual value of the prize, given that it is not paid out all at once?

Exercise 4. Based on the results of the previous tasks, choose the best lottery ticket.

Literature

- Novák, J., *Střední hodnota v úlohách na střední škole*. Učitel matematiky, 2, JČMF, 2024.
- *Herní plán loterií SAZKA* [online] Dostupné z https://static.sazka.cz/kentico-media/sazka/media/content/herni-plany/hp-sazka-od-17-7-24-komplet-sazka.pdf, [cit. 1. 9. 2024]

The portal Math4U is developed within the international projects of the Department of Applied Mathematics (VSB-Technical University of Ostrava) and other institutions in Czechia, Poland, Slovakia, and Spain.
Funded by the European Union