

Optimization

Linear Programming

Linear programming is a mathematical method used to find the best solution to a particular problem. It is a technique that aims to maximize or minimize a linear function under certain constraints, which are also expressed as linear equations or inequations.

This field began to attract the attention of mathematicians only after the First World War. One of the first was Leonid Kantorovich, who was later forced to abandon his work due to the government repression at that time, and eventually out of fear for his life. It was not a good idea to optimize production processes in the Soviet Union, which had a centrally planned economy at the time (for example, in one factory he managed to increase production efficiency to 94%, only to be told that all factories had to increase their efficiency in the same way).

The real turning point in the development of linear programming was the publication of the so-called simplex algorithm for solving these problems in 1947. Its author is the American mathematician George Dantzig, who began working in this field during the Second World War in an attempt to optimize certain processes in the US military. They called it *programming methods using desktop calculators*. In his first technical lecture on the subject, he talked about *programming in a linear structure*, which was subsequently shortened to just *linear programming*. The word *programming* is a relic of military terminology referring to the planning or scheduling of training, logistics, or team deployment.

We will illustrate the principles with the following simple examples.

Optimizing Production in a Roasting Plant

Exercise 1. Berenika and Peter opened a new café with a roastery, where among other things they started to produce two blends of coffee: summer and exotic. The summer blend is made of 40% sweet Ethiopian coffee beans and 60% juicy coffee beans from Peru. The exotic blend is made from the same coffee beans, but in a 3:1 ratio (this time with more Ethiopian coffee beans). There are 90 kg of Ethiopian coffee and 70 kg of Peruvian coffee. A kilo of the summer blend sells for 650 CZK and a kilo of the exotic blend sells for 800 CZK. How much of which blend should Berenice and Peter mix from the available coffee beans to maximize their profit?

The Best Parking Lot

Exercise 2. A local developer has decided to buy a factory that produced video cassettes and magnetophone tapes. The factory is no longer in use, so it will be demolished to make way for a P+R parking lot for cars and a truck park. However, the developer is now solving the problem of how to set the capacity for each type of vehicle. The total available space is 480 m^2 . A parking space for a car takes up 12 m^2 , while for a truck it is 30 m^2 .

However, the planning and building department also requires that the capacity for passenger cars be at least twice as large as for trucks. At the same time, there must be at least 6 parking spaces for trucks.

Determine the optimal number of parking spaces for cars and trucks that will meet all of the conditions above and at the same time maximize the profit from a full parking lot if the payment for each parking space for cars is 100 CZK and for trucks is 400 CZK.