Description of GDP Trends

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Gross Domestic Product (GDP) represents the monetary value of all goods and services newly produced within a specific area during a given period. In macroeconomics, GDP is used as an indicator to assess the performance of national economies. For example, it can be used to compare the economic performance of European Union member states in 2023 (see Figure 1).

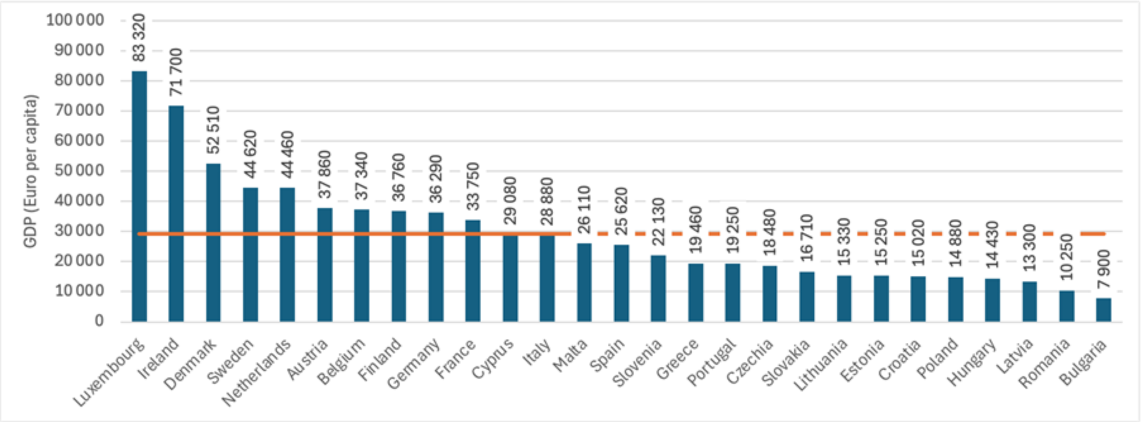

Figure 1: Fig. 1: Comparison of GDP among EU Member States in 2023 (The orange line indicates the EU's average GDP per person, equivalent to €29,280.)

Another way to use GDP is to track a country's performance over time. For example, we can examine the GDP per person for the European Union from 2020 to 2023 when the EU had 27 member states (see Figure 2). The time series illustrating this trend is shown in Figure 2. *(Data source: [2])*

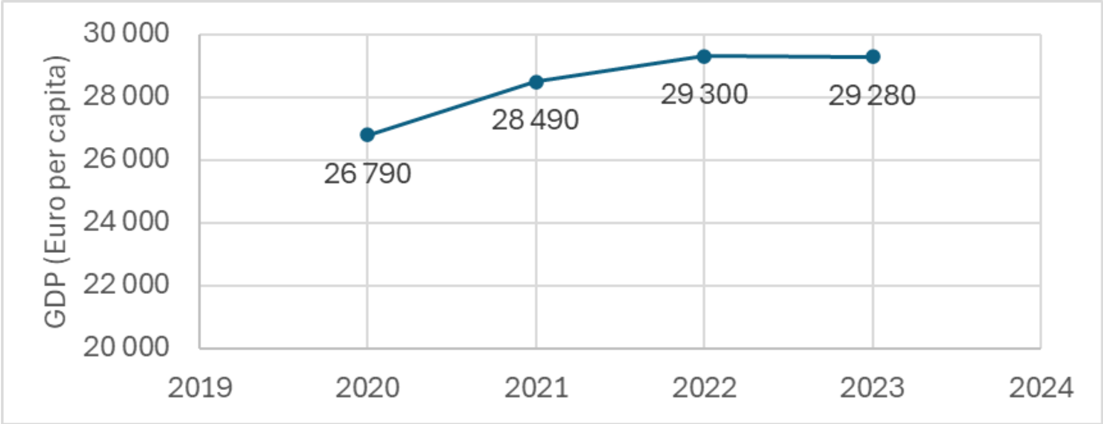

Figure 2: Fig. 2: GDP Trends in EU per Person from 2020 to 2023

A time series showing GDP trends is referred to as an interval-based time series. The data in such a series depend on the length of the analyzed interval (in this case, on the length of the given year).

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Let the values $y_1,\ y_2,\ \ldots\ y_n$ of the time series correspond to the time points $t_1,\ t_2,\ \ldots\ t_n$.

The primary characteristic used to describe this time series is its mean.

The **Mean of an Interval-Based Time Series ($\bar{y}$)** is calculated as a simple arithmetic mean:

$$\bar{y} = \frac{y_1 + y_2 + \cdots + y_n}{n}. \tag{1}$$

Besides the mean, we are often interested in the basic measures of time series dynamics, which help characterize the fundamental features of their behavior.

The **Absolute Change (Δy_t)** is the simplest measure of change in a time series, which tells us "by how much" the series changed between time points:

$$\Delta y_t = y_t - y_{t-1}, \quad t = 2, 3, \ldots, n. \tag{2}$$

The **Average Absolute Change ($\bar{\Delta}$)** indicates the average change between two measurements during the observed period.

The sum of absolute changes represents the total change in the time series over observed period ("by how much" the time series changed from t_1 to t_n):

$$\Delta y_2 + \Delta y_3 + \cdots + \Delta y_n = (y_2 - y_1) + (y_3 - y_2) + \cdots + (y_n - y_{n-1}) = y_n - y_1.$$

Therefore, the average absolute change is calculated as the arithmetic mean of absolute changes:

$$\bar{\Delta} = \frac{\Delta y_2 + \Delta y_3 + \cdots + \Delta y_n}{n-1} = \frac{y_n - y_1}{n-1}. \qquad (3])$$

Notice, this calculation only requires the initial value y_1, the final value y_n, and the number of values n.

Growth Coefficients (Growth Rates, k_t) indicate "how many times" the series changed between time points:

$$k_t = \frac{y_t}{y_{t-1}}, \quad t = 2, 3, \ldots, n. \tag{4}$$

The **Average Growth Coefficient ($\bar{k}$)** indicates the average factor by which the series changed between measurements during the observed period.

In this case, the overall growth coefficient ("how many times" the time series changed between times t_1 a t_n) is calculated not as the sum but as the product of the individual growth coefficients:

$$k_2 \cdot k_3 \cdot \cdots \cdot k_n = \frac{y_2}{y_1} \cdot \frac{y_3}{y_2} \cdot \cdots \cdot \frac{y_n}{y_{n-1}} = \frac{y_n}{y_1},$$

Therefore, the average growth coefficient is determined as the geometric mean of the individual growth coefficients:

$$\bar{k} = \sqrt[n-1]{k_2 \cdot k_3 \cdot \cdots \cdot k_n} = \sqrt[n-1]{\frac{y_n}{y_1}}. \tag{5}$$

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As with the average absolute change, only the initial value y_1, final value y_n, and the number of values n are needed for this calculation.

The **Relative Change (δ_t)**: If we want to know 'by what percentage' the time series changed between individual time points, we use relative changes, which can easily be determined using growth coefficients:

$$\delta_t = \frac{\Delta y_t}{y_{t-1}} \cdot 100 = \frac{y_t - y_{t-1}}{y_{t-1}} \cdot 100 = \left(\frac{y_t}{y_{t-1}} - 1\right) \cdot 100 = (k_t - 1) \cdot 100, \quad t = 2, 3, \ldots, n \tag{6}$$

For example, if the price of a product increases by a factor of 1.5, the percentage increase is $50\%(= (1.5-1)\cdot 100)$.

The **Average Relative Change ($\bar{\delta}$)**

It indicates "by what percentage" the time series changed on average during the period between two measurements within the observed time period can then be easily calculated using the average growth coefficient:

$$\bar{\delta} = (\bar{k} - 1) \times 100. \tag{7}$$

Note The sum (or product) of individual relative changes is not equal to the total relative change ("by what percentage" the time series changed between times t_1 and t_n). Therefore, the arithmetic or geometric mean of individual relative changes cannot be used to calculate the average relative change.

Now, let us attempt a basic description of the time series representing GDP trends (in euros per person), as shown in Figure 2.

Exercise 1. Determine the average annual GDP (in euros per person) of the European Union for the years 2020 to 2023.

Exercise 2. Determine the annual GDP changes (in euros per person) of the European Union from 2020 to 2023 and the corresponding average annual GDP change for this period.

Exercise 3. Determine the annual GDP growth rates of the European Union from 2020 to 2023 and the corresponding average GDP growth rate for this period.

Exercise 4. Determine the annual relative GDP growth rates of the European Union from 2020 to 2023 and the corresponding average relative GDP growth rate for this period.

Literature

- *Real GDP per Capita* [online], Eurostat, 2024, Available at: https://ec.europa.eu/eurostat/databrowser/view/sdg_0 [Accessed: 2024-12-18].
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