MATH4U

Math4You

2023–2025

GDP growth rate

Gross Domestic Product (GDP) is a monetary expression of the total value of goods and services newly created in a given period in a given territory. GDP is used in macroeconomics as an indicator for determining the performance of national economies. The figure shows the gross domestic product per capita of each country in the European Union in 2023.
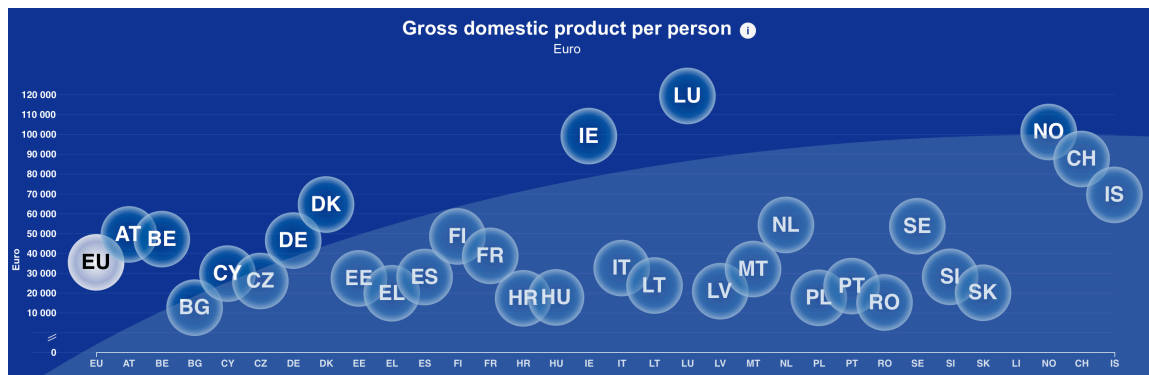

Figure 1: GDP per capita of individual EU countries in 2023

Recall that for n values of $x_1, x_2, \ldots x_n \in \mathbb{R}$ the **arithmetic mean** is

$$\overline{x}_A = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

and the **geometric mean** is

$$\overline{x}_G = \sqrt[n]{x_1 \cdot x_2 \cdot \cdots \cdot x_n}.$$

It holds that $\overline{x}_A \geq \overline{x}_G$, equality occurs if $x_1 = x_2 = \cdots = x_n$.

In this assignment, we will consider data that show the evolution of GDP per capita in the EU between the years 2003 and 2022. For these values, we will be interested in the **annual growth rate**

$$y_i = x_i - x_{i-1},$$

where x_i, x_{i-1} are the GDP values in two consecutive years.

The **relative annual increase** is the annual increase relative to the original value. It is calculated as the fraction

$$\frac{x_i - x_{i-1}}{x_{i-1}}.$$

To characterize data with a certain trend, we often need to calculate the growth rate, for which it is necessary to use the geometric mean.

The **growth coefficient** is the ratio of two consecutive values x_{i-1}, x_i, i.e.

$$z_i = \frac{x_i}{x_{i-1}}.$$

We define the **average growth coefficient** $\overline{z}_G$ as the geometric mean of the individual growth coefficients for a given period

$$\overline{z}_G = \sqrt[n]{z_1 \cdot z_2 \cdot \cdots \cdot z_n}.$$

This relationship can be modified by truncating the fractions below the square root

$$\overline{z}_G = \sqrt[n]{\frac{x_2}{x_1} \cdot \frac{x_3}{x_2} \cdot \cdots \cdot \frac{x_n}{x_{n-1}}} = \sqrt[n]{\frac{x_n}{x_1}}.$$

From here we can see that when calculating the average growth factor, we only need to work with the first and last values of our sequence.

We then call the value $\overline{z}_G - 1$ the **average growth rate**.

The table shows the GDP per capita values in European Union between 2003 and 2022 (from 2020 for 27 countries).

Year	GDP [Euro/capita]
2003	20 330
2004	21 180
2005	22 010
2006	23 200
2007	24 550
2008	25 260
2009	24 050
2010	24 900
2011	25 660
2012	25 770
2013	26 010
2014	26 580
2015	27 500
2016	28 190
2017	29 320
2018	30 290
2019	31 310
2020	30 050
2021	32 520
2022	35 280

Note Data from the table can be copied from the Eurostat dataset https://ec.europa.eu/eurostat/databrowser/view/NAM and processed in a spreadsheet.

Exercise 1. Calculate the annual GDP per capita growth rate.

Solution. The annual growth rate of GDP per capita is the difference between two consecutive values of x_i. The annual growth rate for 2003 is then

$$x_2 - x_1 = 21\,280 - 20\,330 = 850.$$

The annual growth rates in the following years are shown in the table below.

Year	GDP [Euro/capita]	Annual growth [Euro/capita]
2003	20 330	
2004	21 180	850
2005	22 010	830
2006	23 200	1 190
2007	24 550	1 350
2008	25 260	710
2009	24 050	−1 210
2010	24 900	850
2011	25 660	760
2012	25 770	110
2013	26 010	240
2014	26 580	570
2015	27 500	920
2016	28 190	690
2017	29 320	1 130
2018	30 290	970
2019	31 310	1 020
2020	30 050	−1 260
2021	32 520	2 470
2022	35 280	2 760

When using a spreadsheet, you can use the command for the difference of values in two cells and copy it to the whole column.

Exercise 2. Calculate the relative annual GDP per capita growth (annual growth relative to the initial value).

Solution. To calculate relative GDP growth, we divide the value of annual GDP growth by the original value. The relative growth rate for 2003 will then be

$$\frac{x_2 - x_1}{x_1} = \frac{21\,180 - 20\,330}{20\,330} = \frac{850}{20\,330} \doteq 0{,}0418.$$

That's $4{,}18\,\%$ in percent. Relative annual growth in the other years is then calculated in the same way.

Year	GDP [Euro/capita]	Annual growth [Euro/capita]	Relative annual growth [%]
2003	20 330		
2004	21 180	850	4,18
2005	22 010	830	3,92
2006	23 200	1 190	5,41

Year	GDP [Euro/capita]	Annual growth [Euro/capita]	Relative annual growth [%]
2007	24 550	1 350	5,82
2008	25 260	710	2,89
2009	24 050	−1 210	−4,79
2010	24 900	850	3,53
2011	25 660	760	3,05
2012	25 770	110	0,43
2013	26 010	240	0,93
2014	26 580	510	2,19
2015	27 500	920	3,46
2016	28 190	690	2,51
2017	29 320	1 130	4,01
2018	30 290	970	3,31
2019	31 310	1 020	3,37
2020	30 050	−1 260	−4,02
2021	32 520	2 470	8,22
2022	35 280	2 760	8,49

Exercise 3. Calculate the average relative annual increase in GDP per capita.

Solution. The average relative annual GDP growth rate is given by the arithmetic mean of relative GDP growth rates and is equal to $2,99\,\%$.

This result is obtained in a spreadsheet using the AVERAGE or AVERAGEA command.

Exercise 4. Calculate the annual GDP per capita growth rate.

Solutin. The annual GDP growth rate is obtained by adding the relative growth rate to $100\,\%$ or by using the relation $z_i = \frac{x_i}{x_{i-1}}$. Thus, for 2003, we have

$$z_2 = \frac{x_2}{x_1} = \frac{21\,180}{20\,330} \doteq 1,0418$$

in percentages, i.e. $104,18\,\%$. In subsequent years, we calculate the annual growth rates in the same way.

Year	GDP [Euro/capita]	Annual growth [Euro/capita]	Relative annual growth [%]	Annual growth coefficient[%]
2003	20 330			
2004	21 180	850	4,18	104,18
2005	22 010	830	3,92	103,92
2006	23 200	1 190	5,41	105,41
2007	24 550	1 350	5,82	105,82
2008	25 260	710	2,89	102,89
2009	24 050	−1 210	−4,79	95,21
2010	24 900	850	3,53	103,53
2011	25 660	760	3,05	103,05
2012	25 770	110	0,43	100,43
2013	26 010	240	0,93	100,93
2014	26 580	510	2,19	102,19

Year	GDP [Euro/capita]	Annual growth [Euro/capita]	Relative annual growth [%]	Annual growth coefficient[%]
2015	27 500	920	3,46	103,46
2016	28 190	690	2,51	102,51
2017	29 320	1 130	4,01	104,01
2018	30 290	970	3,31	103,31
2019	31 310	1 020	3,37	103,37
2020	30 050	−1 260	−4,02	95,98
2021	32 520	2 470	8,22	108,22
2022	35 280	2 760	8,49	108,49

Exercise 5. Calculate the average GDP per capita growth rate.

Solution. To obtain the average GDP growth rate, it is sufficient to use the initial and final values:

$$\overline{z}_G = \sqrt[19]{\frac{35\,280}{20\,330}} \doteq 1{,}0294,$$

in percentages, $102{,}94\,\%$. The average GDP growth rate per capita is therefore $2{,}94\,\%$. In a spreadsheet, the GEOMEAN geometric mean command can be used.

Exercise 6. Estimate the GDP per capita in 2022 based on the 2003 value using a) the average relative annual growth, b) the average growth rate.

Solution. Note that the values of the average relative growth rate and the average growth rate are different. We will use the exact value obtained in the spreadsheet for the calculation.

a) The average relative growth rate is about $2{,}99\,\%$, so to get the following value we need to multiply the original value by $1{,}0299$. To get an estimate of the GDP per capita value, we need to calculate $20\,330 \cdot 1{,}0299^{19} \doteq 35\,615$ Euros per capita.

b) Using the geometric mean of GDP growth coefficient per capita, we obtain an exact value of $20{,}330 \cdot ,0294^{19} \doteq 35{,}28$

Literature

- Eurostat. *HDP per person* [online]. Dostupné z https://ec.europa.eu/eurostat/cache/visualisations/country-bubbles [cit. 9.,8.,2023].
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- Český statistický úřad. *Hrubý domácí produkt - metodika.* Dostupné z https://www.czso.cz/csu/czso/hruby_doma hdp- [cit. 9.,8.,2023]