The Curious Warehouse Manager

When we solve purely mathematical problems, we get exact results. However, when we use mathematics to solve problems in the world around us, we rarely achieve absolute precision in the answer. Approximation is sometimes the result of a simplification of the real situation in our minds. Sometimes the input data are approximated (e.g. we can only measure lengths or time with limited accuracy) or an absolutely exact result is realistically unattainable and must be rounded off.

Rounding to a given number of significant digits is often used in practice (and in the following problems). We round a positive real number r to n significant digits as follows:

- We express r in the form $a \cdot 10^b$, where $a \in \mathbb{R}, a \in \langle 1, 10 \rangle$ and $b \in \mathbb{Z}$, and then we round the number a to $n-1$ decimal places according to the standard rules for rounding.
- E.g. numbers $r = 31.258\,16$ and $s = 0.023\,123\,6$ we round to four valid digits as follows:

$$r = 31.258\,16 = 3.125\,816 \cdot 10^1 \quad \doteq \quad 3.126 \cdot 10^1 = 31.26$$
$$s\ = 0.023\,123\,6 = 2.312\,36 \cdot 10^{-2} \quad \doteq \quad 2.312 \cdot 10^{-2} = 0.023\,12.$$

Notably, rounding input data can have surprising consequences for the accuracy of the result, for example when solving equations, as we will see in the following series of problems.

Exercise 1. The manager of the pharmaceutical warehouse received an invoice for two types of ordered vaccines. A total of $401,950$ CZK was paid for the delivery of 597 packages of Ixodinum vaccines against encephalitis and 386 packages of Nopolio vaccines against polio. However, during initial inspection, 86 packages of Ixodinum vaccine and 19 packages of Nopolio vaccine were found to be expired and had to be returned. A total of $39,600$ CZK was refunded for the expired medicines. Out of curiosity, the manager wants to calculate the purchase price of one package of both vaccines. However, he does not have a calculator or a mobile phone on hand, so he settles for an approximate solution. He rounds all the figures he knows to one significant digit before calculating.
How much will his result differ from the actual purchase price? For both types of vaccines, determine the absolute difference between the calculated and actual prices, as well as the relative error expressed as a percentage.

Exercise 2. After a few months, another delivery arrived at the warehouse, namely 504 packages of Antiflu vaccines against influenza and 81 packages of Kontradift vaccines against diphtheria. $198,900$ CZK was paid for this delivery. During initial inspection, 98 packages of Antiflu and 18 packages of Contradift were found to be expired. A total of $40,700$ CZK was refunded.
The warehouse manager repeated his procedure and calculated the approximate purchase price of the two drugs off the top of his head. This time, however, he was surprised. What was the reason for his surprise and how much did his result differ from the actual prices?

Exercise 3. Graphically represent the systems of equations from the previous two problems using appropriate software. Explain the difference in the accuracy of the results of the two exercises by comparing their graphs.

Literature

- Biermann K., Grötschel M., Lutz-Westphal B. (2010). *Besser als Mathe: Moderne angewandte Mathematik aus dem MATHEON zum Mitmachen.* Berlin: Vieweg+Teubner.