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Collapse of Cod Fisheries

Keywords: functions, quadratic function, quadratic equation

Coastal states have a vast wealth of fish in the oceans within their grasp. This wealth is seemingly endless and stable. However, people have learned some bitter lessons that this is not the case. One significant lesson dates back to 1992. The Gulf off Newfoundland had always been rich in cod (*Gadus morhua*, Atlantic cod). A boat that came to fish there never left without a rich catch. But over time, the situation began to change. In the late 1980s, biologists called for a 50% reduction in fishing to avoid plundering the fishery. However, because a reduction in fishing would drag the area into recession, the government did not decide to impose limits. Unfortunately, nature follows its own laws. Gradually, the situation reached the point where halting fishing was inevitable. The cod population fell to just one percent of its original level. A moratorium on fishing was therefore declared. Initially, the moratorium was to last two years. However, the small cod population did not recover substantially. Therefore, the restrictions have lasted much longer than originally anticipated. Despite some hope of easing restrictions in 2015, the allowable harvest rate was reduced again in 2018 after the population collapsed again. The moratorium on fishing resulted in job losses for 35,000 fishermen and fish processing factory workers. This had huge economic and sociological impacts on the entire region.

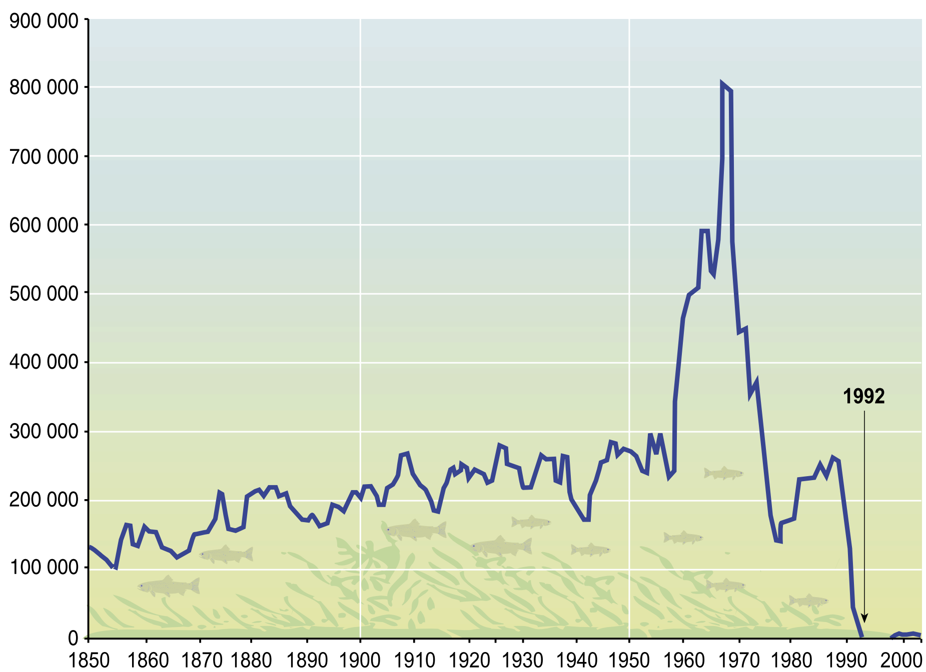

Figure 1: The graph shows the evolution of the Newfoundland cod fishery in tons of fish. Source: Millennium Ecosystem Assessment

It should be added that the case described above is not unique. Simultaneously with the collapse of the Newfoundland fisheries, a similar situation occurred in five other Canadian fisheries where a moratorium on fishing was issued in 1993 (Southern Grand Bank, St. Pierre Bank, Northern Gulf of St. Lawrence, Southern Gulf of St. Lawrence, Eastern Scotian Shelf). And have you read Steinbeck's 1945 novel *Cannery Row*? It describes life around a sardine factory in California. Shortly after the novel was

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published, the fishery began to collapse due to unsustainable fishing, and commercial fishing had to be banned in 1967.

Modeling Population Growth

In order to prevent fisheries collapses and to be able to realistically and effectively model population growth in nature, effective and time-tested mathematical models have been developed. One simple yet reasonably accurate model describes the population growth rate using a quadratic function:

$$f(N) = rN\left(1 - \frac{N}{K}\right),$$

where N is the population size, $f(N)$ is the population growth rate, and r and K are constants. The constant K is called the carrying capacity of the environment. The constants r and K determine the reproductive capabilities of the population and the impact of the environment on the population. These constants have also given names to the *r*/*K* selection theory which describes two basic life strategies that help populations in nature to establish and thrive successfully. Populations that qualify as *r*-strategists are able to reproduce rapidly. They do not care much for their offspring and compensate for care by abundance. These populations have a large value of the constant r. In contrast, *K*-strategists have few offspring, but care for them and can cope better with environmental changes. Therefore, their population sizes are closer to the carrying capacity of the environment than is the case for *r*-strategists.

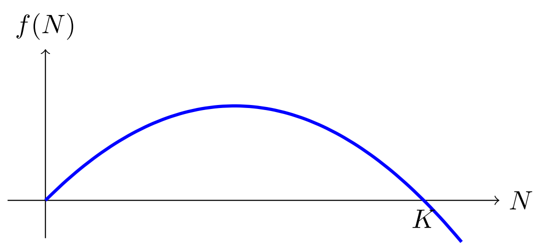

Figure 2: Population growth rate as a function of population size.

The growth rate indicates how much the population size increases per unit time. If it is zero, the population size does not change. If the growth rate is positive and numerically large, the population size grows rapidly. If the growth rate is negative, the population size decreases and the population dies out. The graph of the function modelling growth rate is shown in the figure. This model captures the well-known facts that a population of small size reproduces slowly (a small population has few individuals and hence few individuals capable of reproduction). The model also captures the fact that a larger population reproduces faster, but only to a certain extent that the carrying capacity of the environment allows.

Problems

Consider a hypothetical population exposed to harvesting. We will measure the population size in appropriate units. This can be in numbers of individuals, in thousands of individuals, in tons, and so on. For example, consider the parameters $K = 1000$ and $r = 0.1$. That is, the size of the population that

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can sustain in the environment is 1000, and a small population that does not suffer from intraspecific competition grows at 10% of its current size per unit time.

Problem 1. Determine the population size N_* which guaranties the maximum growth rate. Find this maximum growth rate. We will henceforth denote this value by h_*, as it is also the maximum theoretical possible harvesting rate (also called harvesting intensity). The value N_* is the population size at this maximum rate.

Problem 2. Determine how many times the population growth rate decreases if the population size drops from the size N_*, which allows the maximum possible harvesting intensity, to one percent of this size. This is the value to which the harvest would have to be reduced to prevent $>$ further decline. (In practice, however, we would want population recovery, and therefore, the restriction specified in this step alone is not sufficient.)

Problem 3. Assume the careful fishing at 80 percent of the maximum sustainable harvest h_*. Even in this case a caution is necessary. If the population is too small, it cannot cope with fishing. Determine what is the minimum size of the population capable of coping with fishing at the rate equal to 80 percent of h_* without collapsing.

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