Math4You

2023–2025

Collapse of Cod Fisheries

Coastal states have a vast wealth of fish in the oceans within their grasp. This wealth is seemingly endless and stable. However, people have learned some bitter lessons that this is not the case. One significant lesson dates back to 1992. The Gulf off Newfoundland had always been rich in cod (*Gadus morhua*, Atlantic cod). A boat that came to fish there never left without a rich catch. But over time, the situation began to change. In the late 1980s, biologists called for a 50% reduction in fishing to avoid plundering the fishery. However, because a reduction in fishing would drag the area into recession, the government did not decide to impose limits. Unfortunately, nature follows its own laws. Gradually, the situation reached the point where halting fishing was inevitable. The cod population fell to just one percent of its original level. A moratorium on fishing was therefore declared. Initially, the moratorium was to last two years. However, the small cod population did not recover substantially. Therefore, the restrictions have lasted much longer than originally anticipated. Despite some hope of easing restrictions in 2015, the allowable harvest rate was reduced again in 2018 after the population collapsed again. The moratorium on fishing resulted in job losses for 35,000 fishermen and fish processing factory workers. This had huge economic and sociological impacts on the entire region.

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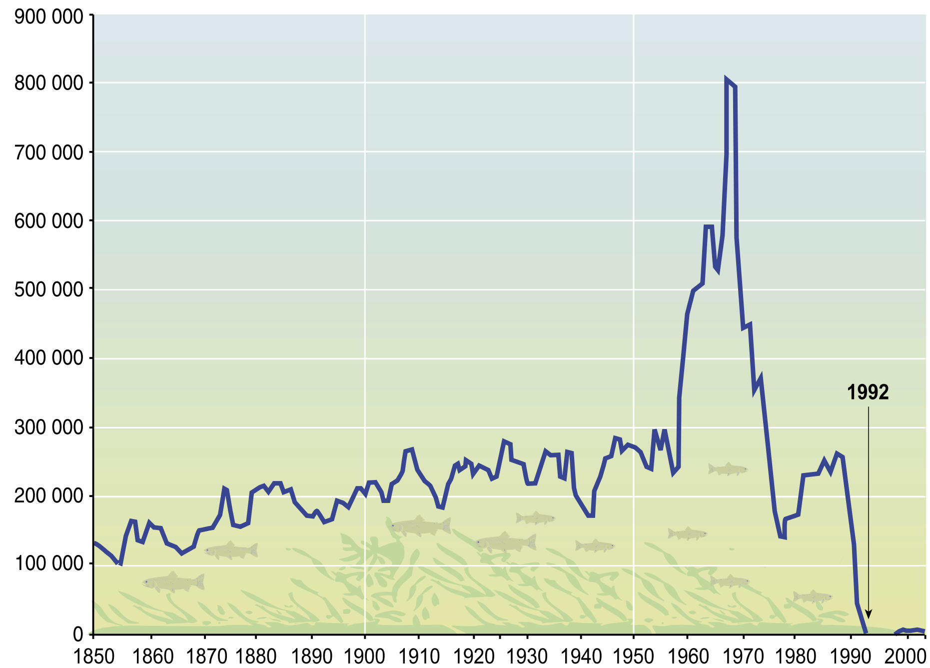

Figure 1: The graph shows the evolution of the Newfoundland cod fishery in tons of fish. Source: Millennium Ecosystem Assessment

It should be added that the case described above is not unique. Simultaneously with the collapse of the Newfoundland fisheries, a similar situation occurred in five other Canadian fisheries where a moratorium on fishing was issued in 1993 (Southern Grand Bank, St. Pierre Bank, Northern Gulf of St. Lawrence, Southern Gulf of St. Lawrence, Eastern Scotian Shelf). And have you read Steinbeck's 1945 novel *Cannery Row*? It describes life around a sardine factory in California. Shortly after the novel was published, the fishery began to collapse due to unsustainable fishing, and commercial fishing had to be banned in 1967.

Modeling Population Growth

In order to prevent fisheries collapses and to be able to realistically and effectively model population growth in nature, effective and time-tested mathematical models have been developed. One simple yet reasonably accurate model describes the population growth rate using a quadratic function:

$$f(N) = rN\left(1 - \frac{N}{K}\right),$$

where N is the population size, $f(N)$ is the population growth rate, and r and K are constants. The constant K is called the carrying capacity of the environment. The constants r and K determine the reproductive capabilities of the population and the impact of the environment on the population. These constants have also given names to the *r*/*K* selection theory which describes two basic life strategies that help populations in nature to establish and thrive successfully. Populations that qualify as *r*-strategists are able to reproduce rapidly. They do not care much for their offspring and compensate for care by abundance. These populations have a large value of the constant r. In contrast, *K*-strategists have few offspring, but care for them and can cope better with environmental changes. Therefore, their population sizes are closer to the carrying capacity of the environment than is the case for *r*-strategists.

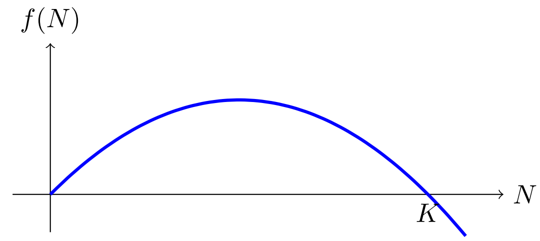

Figure 2: Population growth rate as a function of population size.

The growth rate indicates how much the population size increases per unit time. If it is zero, the population size does not change. If the growth rate is positive and numerically large, the population size grows rapidly. If the growth rate is negative, the population size decreases and the population dies out. The graph of the function modelling growth rate is shown in the figure. This model captures the well-known facts that a population of small size reproduces slowly (a small population has few individuals and hence few individuals capable of reproduction). The model also captures the fact that a larger population reproduces faster, but only to a certain extent that the carrying capacity of the environment allows.

Problems

Consider a hypothetical population exposed to harvesting. We will measure the population size in appropriate units. This can be in numbers of individuals, in thousands of individuals, in tons, and so on. For example, consider the parameters $K = 1000$ and $r = 0.1$. That is, the size of the population that can sustain in the environment is 1000, and a small population that does not suffer from intraspecific competition grows at 10% of its current size per unit time.

Problem 1. Determine the population size N_* which guaranties the maximum growth rate. Find this maximum growth rate. We will henceforth denote this value by h_*, as it is also the maximum theoretical possible harvesting rate (also called harvesting intensity). The value N_* is the population size at this maximum rate.

Solution. Function

$$f(N) = rN\left(1 - \frac{N}{K}\right),$$

which describes growth is a quadratic function and its graph is a parabola. This graph is only meaningful for $N \geq 0$.

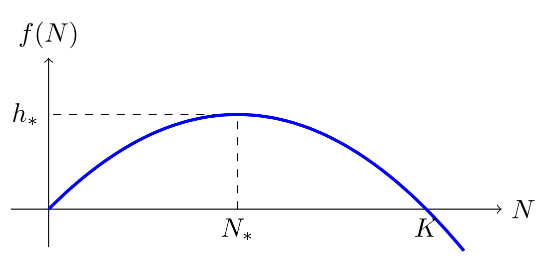

Figure 3: Population growth rate as a function of population size, indicating the maximum possible harvesting intensity h_* and the corresponding stable population size N_*.

Since the function is given in the form of a product of the root Factors, we see that the roots are $N = 0$ and $N = K$. The function takes its maximum at the vertex of the parabola, i.e., for $N_* = \frac{K}{2} = 500$. The function value is

$$h_* = f(N_*) = r\frac{K}{2}\left(1 - \frac{\frac{K}{2}}{K}\right) = \frac{rK}{4}$$

and for the given values of the constants K and r we get

$$h_* = \frac{0.1 \cdot 1000}{4} = 25.$$

Comparing this to the carrying capacity of the environment ($K = 1000$), we see that this value is 2.5 percent of the carrying capacity of the environment. Since the population stabilizes at half of the carrying capacity when harvesting is at this rate, this means that the fishing proceeds at such a rate that 5 percent of the current population is harvested per unit time.

Problem 2. Determine how many times the population growth rate decreases if the population size drops from the size N_*, which allows the maximum possible harvesting intensity, to one percent. This is the value to which the harvest would have to be reduced to prevent further decline. (In practice, however, we would want population recovery, and therefore, the restriction specified in this step alone is not sufficient.)

Solution. Let N_2 be the size of the population after the decline. Then

$$N_2 = 0.01N_* = 0.01\frac{K}{2}$$

and we get

$$f(N_2) = r \cdot 0.01\frac{K}{2}\left(1 - \frac{0.01\frac{K}{2}}{K}\right) = 0.004975 \cdot r\,K$$

and

$$\frac{f(N_2)}{f(N_*)} = \frac{0.004975rK}{0.25rK} \approx 0.02.$$

If the population size drops to one percent, the harvesting intensity must be reduced to two percent of the original intensity to avoid further decline.

Problem 3. Assume the careful fishing at 80 percent of the maximum sustainable harvest h_*. Even in this case a caution is necessary. If the population is too small, it cannot cope with fishing. Determine what is the minimum size of the population capable of coping with fishing at the rate equal to 80 percent of h_* without collapsing.

Solution. According to the assignment, we need to solve the equation

$$rN\left(1 - \frac{N}{K}\right) = 0.8\frac{rK}{4}.$$

We can expand the parentheses and move all terms to one side to obtain the form

$$-\frac{r}{K}N^2 + rN - 0{,}8\frac{rK}{4} = 0.$$

For $r = 0.1$ and $K = 1000$ we get

$$-0.0001N^2 + 0.1N - 20 = 0$$

which can be rewritten as

$$N^2 - 1000N + 200000 = 0$$

The roots of this quadratic equation are

$$N_{1,2} = \frac{1000 \pm \sqrt{1000^2 - 4 \cdot 200000}}{2}$$

and hence

$$N_1 \approx 276$$

and

$$N_2 \approx 724.$$

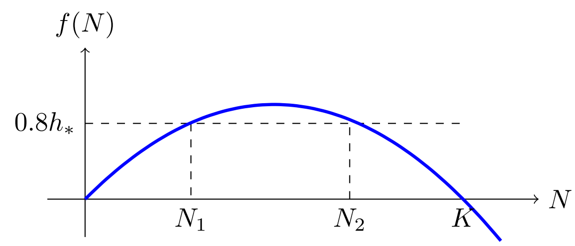

Figure 4: Population growth rate as a function of time, with the set harvesting intensity and states where the harvesting intensity is the same as the natural population growth rate plotted.

The figure shows the parabola defining the growth rate, the horizontal line defining the harvesting rate and the intersections N_1 and N_2. For population sizes smaller than N_1 harvesting exceeds the growth. In this situation, population growth is not capable of compensating the harvesting rate. The population is overfished, declines, and collapses. To set fishing at 80 percent of the maximum sustainable harvest, it is necessary to wait until the population grows to a size of $N_1 = 276$. This value is slightly more than half of N_*, i.e., more than half the value at which the population stabilizes at the maximum sustainable harvesting intensity.

This last part shows that after a population collapse, it is not possible to set an earlier sustainable harvesting intensity and hope for a spontaneous recovery of the population. The population must have sufficient growth dynamics to cope with this level of harvesting. It is necessary to wait until the fish population returns to sufficiently large stocks. It is possible to return to the previous harvesting rate only if the population size, which prevents extinction, is achieved.

References and literature

Literature

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