Hyperbolic navigation

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Progress in the field of electrical engineering enabled the development of new navigation systems based on the transmission of electromagnetic waves. An example of such a system is LORAN-C marine navigation, which was developed during World War II in the USA. In this type of navigation, the vessel receives a synchronized signal from a pair of transmitters. The signal from the more distant transmitter is received by the vessel later, so the signal delay determines the difference between the distances of the vessel from the first and second transmitter.

The set of points that have a constant difference in distances from two given fixed points is a hyperbola. Thus, the vessel is located on a hyperbola, whose foci are the transmitters, and which is determined by the difference in the vessel's distances from those transmitters. The signal delay from another pair of stations then determines the second hyperbola on which the vessel must lie. If the vessel lies on both hyperbolas, it lies at their intersection.

Exercise. Three receivers P_1, P_2, and P_3 are deployed in the landscape. The figure captures the distances we know:

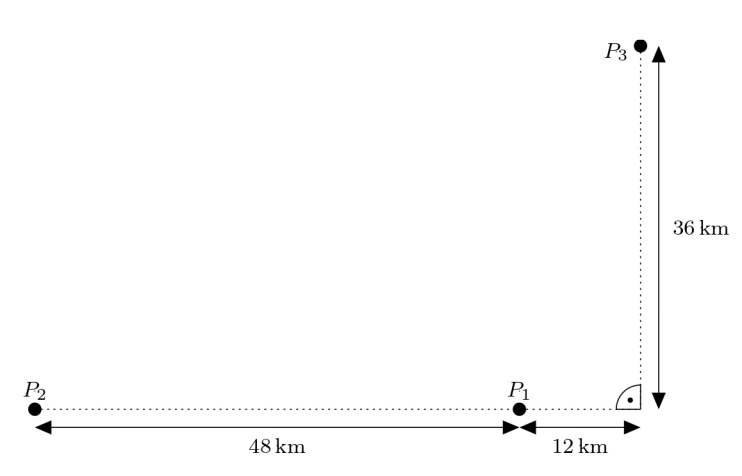

Figure 1: Exercise assignment

Adam's tourist navigation will send a signal to all three receivers. Signal arrives at receivers P_1 and P_3 at the same time and at receiver P_2 80 microseconds later. Where is Adam located? Assume the signal travels 300 000 km per second. Determine the position in a suitably established coordinate system.

Note. If Adam was not equidistant from receivers P_1 and P_3, solving the problem would mean finding the intersection of the branches of two hyperbolas. However, such a calculation would be beyond the scope of high school mathematics.

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Literature

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