Sound

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Sound is a mechanical wave that we perceive by hearing. All people perceive the pitch and duration of a tone in approximately the same way, but the perception of loudness is very subjective. Loudness is determined by the amplitude of the oscillation in the medium through which the sound wave propagates. Since the amplitude of sound waves is not easily measured, the quantities of sound intensity I and sound intensity level L are used to compare loudness objectively.

Sound intensity expresses how much energy sound waves transfer to a unit area perpendicular to the direction of sound propagation per unit time. A healthy ear can detect the smallest sound intensity $I_0 = 10^{-12}\,\text{W/m}^2$ at a frequency of $1000\,\text{Hz}$, which corresponds to the hearing threshold. On the other hand, the sound intensity of $10\,\text{W/m}^2$ is loud enough to correspond to the pain threshold. However, increasing the sound intensity I ten times does not correspond to perceiving the sound ten times louder. Therefore, the sound intensity level L is rather used to express sound loudness, which uses a logarithmic scale in decibels (dB).

Sound intensity level L in decibels is defined by the equation

$$L = 10 \log \frac{I}{I_0},$$

where I is the sound intensity at the given location and $I_0 = 10^{-12}\,\text{W/m}^2$, which corresponds to the hearing threshold. The sound intensity level of $60\,\text{dB}$ corresponds to the loudness of a normal conversation, $90\,\text{dB}$ is the loudness of a lawnmower, and $110\,\text{dB}$ is the loudness of a discotheque.

There is a risk of hearing impairment for long-term listening (even though we are not in pain) of volumes higher than $85\,\text{dB}$. From a volume higher than $100\,\text{dB}$, there is a risk of hearing damage within minutes. Let's explore the relation between sound intensity and sound intensity level, i.e., the loudness perceived by hearing.

Exercise 1. The sound intensity is $1{,}27 \cdot 10^{-3}\,\text{W/m}^2$ while listening to a speaker with a sound power of $20\,\text{W}$ at a distance of $50\,\text{m}$ from it (assume uniform transmission of the sound wave into free half-space). How many decibels do we measure at this location?

Exercise 2. How will the sound intensity level change if there is twice the sound intensity at the location from the previous example, i.e., $2 \cdot 1{,}27 \cdot 10^{-3}\,\text{W/m}^2$?

Exercise 3. From the formula for the sound intensity level, find the value ΔL, by which the sound intensity level L, changes if the sound intensity is doubled from I to $2I$.

Exercise 4. Sound intensity is inversely proportional to the square of the distance from the sound source. By how much does the sound intensity level change if the distance from the sound source is doubled?

Exercise 5. From the formula for the sound intensity level $L = 10 \log \frac{I}{I_0}$, express the sound intensity I.

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Exercise 6. By how many times will the sound intensity increase if the sound intensity level increases by $20\,\mathrm{dB}$?

Literature

1. Kubera, Miroslav; Nečas, Tomáš; Beneš, Vojtěch. *Online učebnice fyziky pro gymnázia - Zvuk.* Available from `https://e-manuel.cz/kapitoly/mechanicke-vlneni/vyklad/zvuk/` [cit. 24.10.2023].

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