Golf Ball Shot

Keywords: differential and integral calculus, optimization, quadratic equation, derivation

Oblique Projectile Motion

Projectile motion is the most general way of setting an object in a homogeneous gravitational field into motion. Suppose that a body (point mass) is thrown obliquely into space without resistance. The initial velocity is $\vec{v}_0$ and the angle between the vector $\vec{v}_0$ and horizontal direction is α. Let us introduce the Cartesian coordinate system with horizontal x-axis and vertical y-axis as shown in the picture. The coordinates of the initial velocity vector are

$$\vec{v}_0 = (v_0 \cos\alpha, v_0 \sin\alpha).$$

The motion of the body is governed by an acceleration due to gravity g directed vertically downwards. The horizontal component of the gravitational acceleration is zero, therefore the motion in the horizontal direction is unaffected by the gravitational field. The vertical component of the motion is affected by the negative acceleration $-g$. Hence, it is a movement with constant (uniform) deceleration and initial velocity $v_0 t \sin\alpha$.

We can use formulas for distance of motion with constant speed and constant acceleration to quantify the coordinates of the point mass. By doing so we get

$$\begin{aligned} x(t) &= v_0 t \cos\alpha, \\ y(t) &= v_0 t \sin\alpha - \frac{1}{2} g t^2. \end{aligned} \tag{1}$$

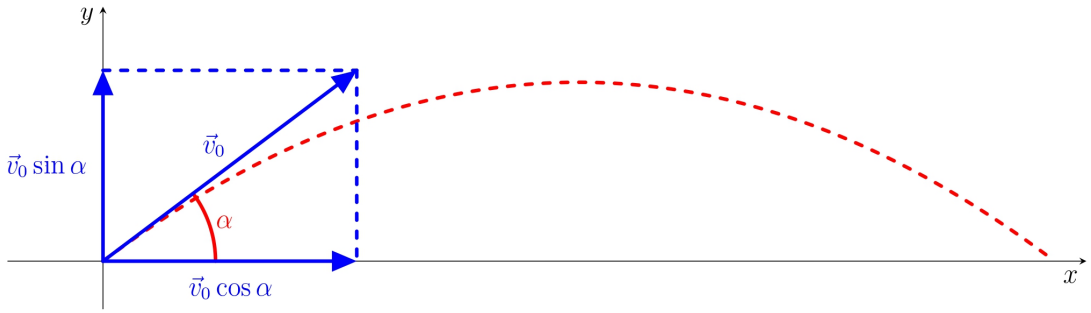

Figure 1: Oblique motion

Motion of a Golf Ball

A golfer hits a ball with an initial velocity v_0. The angle between the initial velocity and the horizontal plane is α. Let's assume that the drag force is negligible. The motion of the ball therefore satisfies the conditions for movement of a projectile launched at an angle in an environment without air resistance.

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Exercise 1. Prove that the trajectory of the golf ball follows a parabolic path.

Exercise 2. Calculate the height of the throw, i.e. the maximal height y_{max} that the launched ball reaches.

Exercise 3. Given constant initial velocity, find the angle α which guarantees maximal distance between the initial and the terminal point of the trajectory.

Literature

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