

Distances on the Earth's surface

Keywords: space geometry, stereometry, planimetry, circle, geography

Which of the routes between Lisbon and Washington shown on the map is shorter?

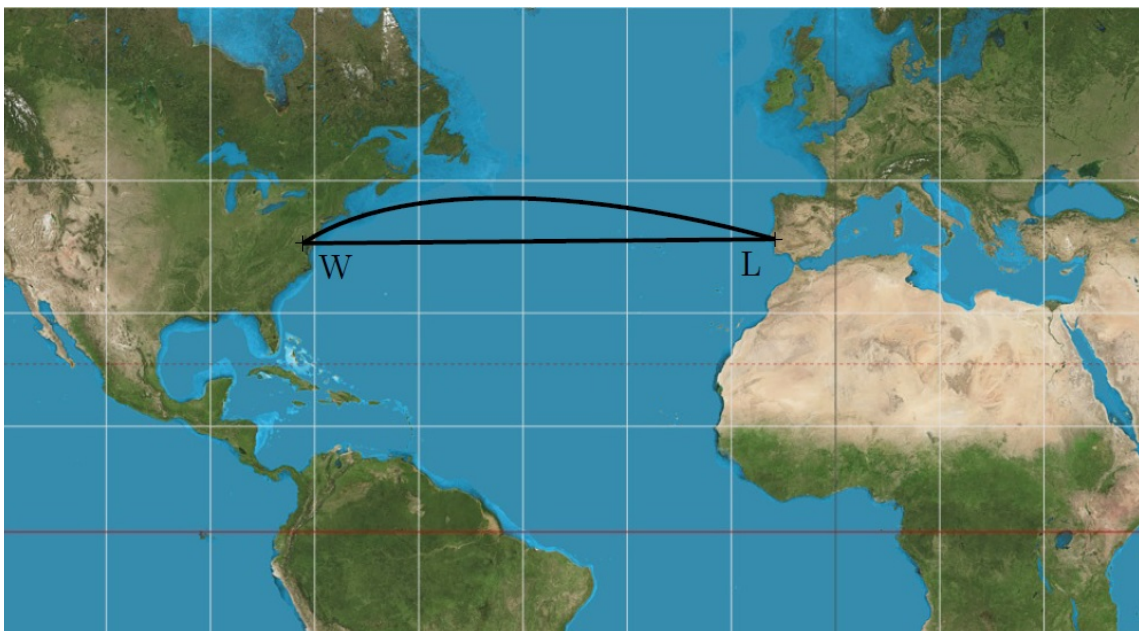


Figure 1: Map

This seemingly simple question has a surprising answer, as you will see in this exercise. The shorter route is the arc, the longer one is the line segment. The reason is the distortion of the distances in the selected representation of the Earth's surface. We see that the line segment LW on the map is approximately parallel to the geographic parallels on the Earth, so it actually corresponds to an arc on a circle that closely resembles a parallel (see the circle k with the center O in the figure).

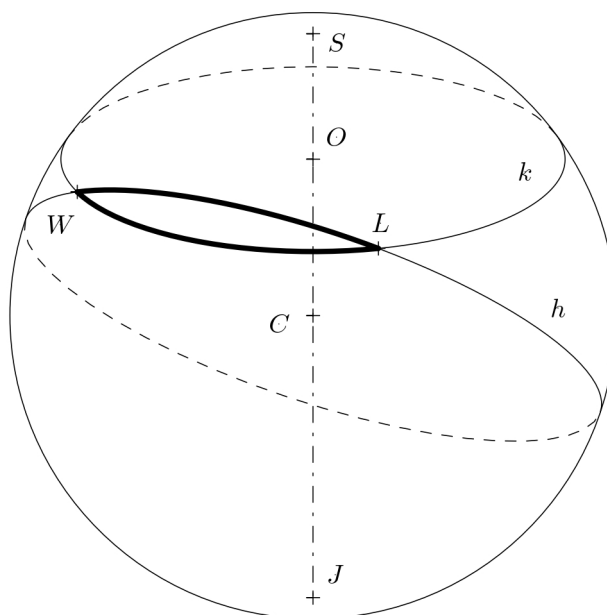


Figure 2: Parallel and great circle

However, on the spherical surface (which we will consider to be the Earth's surface in this task), the shortest distance is another arc. This arc is a part of a circle h whose center C is the center of the Earth. We refer to such paths as *orthodromes* and call all circles with this property *great circles*. How many kilometers do we save by traveling along an orthodrome? The answer to this question has to be calculated.

Dictionary

- *Latitude* of a point on the Earth's surface (expressed in degrees and north/south orientation) is the angle between a straight line that passes through the given point and the center of the Earth and the equatorial plane.
- *Longitude* of a point on the Earth's surface (expressed in degrees and east/west orientation) is the angle between the plane of the meridian that passes through the given point and the plane of the zero meridian.

Exercise. Lisbon and Washington are located at approximately the same parallel (about 39° north latitude). How many kilometers does an airplane save by traveling on an orthodromic path compared to traveling on a parallel path? Lisbon is located at approximately 9° west longitude. Washington is located at 77° west longitude. Let us assume that the Earth is a sphere with center C and radius 6,371 km and that the plane flies at an average altitude of 10 km (take-off and landing are not taken into account). Therefore, in all considerations, we will work with a sphere of radius $\varrho = 6,381$ km.

Literature

- Novák V., Murdych Z. *Kartografie a topografie*. Praha: Státní pedagogické nakladatelství. (1988)
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Sources of figures

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