



Math4You

2023–2025

## Conical cap

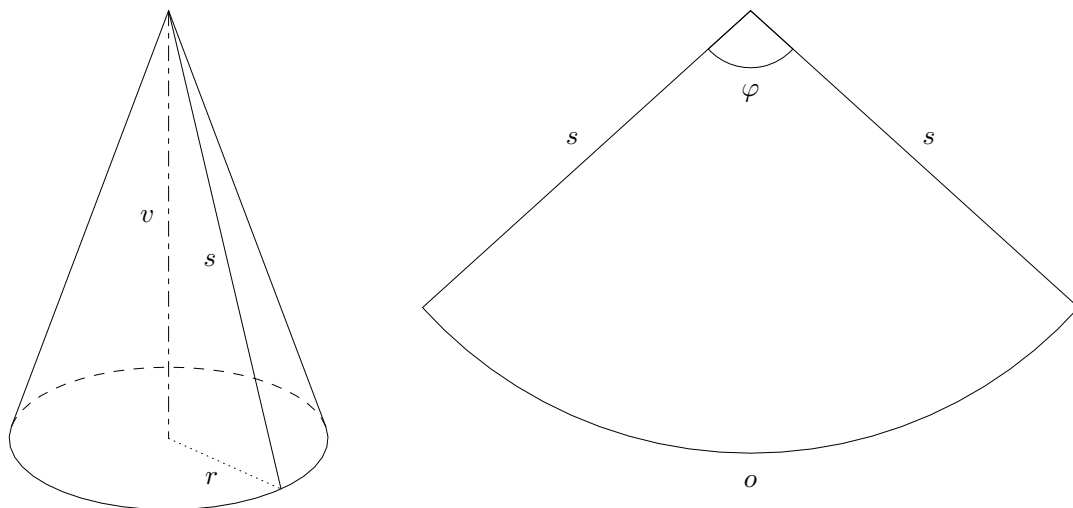
Eight-year-old Annie wants to go to a children's carnival in a white lady costume, which will include a white cone-shaped hat. The parents took the opportunity to practice their geometric imagination with Anna and decided to make the hat with her instead of buying it.

**Exercise.** Anna and her mother found out with a tape measure that the circumference of Anna's head is 52 cm. Together they further agreed that the hat would be 30 cm high. How will they create the hat?

*Solution.* The cap is formed by the lateral surface/face of a cone, where we know the perimeter of the base  $o$  (52 cm) and the height of the cone  $v$  (30 cm). The unrolled lateral face of the cone is then a circular sector with an unknown radius  $s$  (the size of the side of the cone) and an unknown central angle  $\varphi$ . We need to calculate these data. Furthermore, we know that the arc length of a circular sector is equal to the perimeter of  $o$ .

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**Figure 1:** Kuželová čepice

First, we calculate the radius of the base  $r$  from the perimeter of the base and then the length of the side  $s$  using the Pythagorean theorem.

$$r = \frac{o}{2\pi} = \frac{52}{2\pi} \doteq 8,28 \text{ cm}$$

$$s = \sqrt{v^2 + r^2} = \sqrt{30^2 + 8,28^2} \doteq 31,12 \text{ cm}$$

Now we determine the angle  $\varphi$ . First, we calculate the perimeter  $O$  of the whole circle of radius  $s$ , we get

$$O = 2\pi s \doteq 195,53 \text{ cm}.$$

Next, we use the direct proportionality between the arc length of this circle and the corresponding central angle to calculate the angle  $\varphi$ :

$$\varphi = \frac{o}{O} \cdot 360^\circ = \frac{52}{195,53} \cdot 360^\circ \doteq 95^\circ 44'.$$

Annie and her parents create a hat out of a circular sector with an approximate radius of 31 cm and a central angle of approximately  $96^\circ$ .